INSTITUTE OF DISTANCE AND OPEN LEARNING

Gauhati University

H O M E   A S S I G N M E N T

M. A./M.Sc. Mathematics

2013-2014 Session

(2nd Semester)

***Guidelines for Submission*:**

1. Write your name, session, roll number, the topic selected and the title of the answer *clearly on the top.*
2. Each of the two topics given in each paper will be answered as **two essays** of *not more than 500 words each*. There will be negative marking for writing in excess of the word-limit.
3. Each answer (essay) carries a weightage of **10 marks**. (10 marks x 2 essays = 20 marks).
4. Keep a margin of about 1 inch on each side of the page.
5. You can submit the essay written in your own hand-writing on clean A-4 sized paper.
6. In case you prefer to submit type-written answers, make sure that there are no typing errors which will deduct from the overall impression.
7. Do not submit commercially purchased answers as such a practice is deemed to be unfair.
8. Please submit your assignment by **15th May, 2014.**

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201. Complex Analysis (answer any two) $2\times10=20$

1. Define analytic function and harmonic function with suitable examples.
   Let u and v be real-valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Prove that $f : G \to C$ defined by $f(z) = u(z) + i v(z)$ is analytic if and only if u and v satisfy the Cauchy Riemann equations.
2. Prove Residue theorem.
3. Find a necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.
   Show that the transformation
   $$w = \frac{i(1-z)}{(1+z)}$$
   transforms the circle $|z| = 1$ into the real axis of the w-plane and the interior of the circle $|z| < 1$ into upper half of the w-plane.

202. Functional Analysis (answer any two) $2\times10=20$

1. The role of continuous real-valued functions on [0,1] on the Banach space theory
   a. Discuss the space C[0,1] with respect to the norm $\| f \| = \max_{o \le t \le 1} | f(t) |$
   b. Discuss the space C[0,1] with respect to the norm $\| f \| = \int_0^1 | f(t) | dt$
   c. Discuss the space C[0,1] as a Banach algebra
2. The role of ℓ_p, $1 \le p < \infty$ spaces on banach space theory
   a. Discuss the space ℓ_1 and ℓ_2
   b. Discuss the space ℓ_p, $p > 1$ and deduce their fundamental properties
3. Hahn-Banach Theorem, open Mapping Theorem, closed Graph Theorem and their fundamental properties
   a. Describe the above theorems
   b. Describe some fundamental applications on banach space theory

203. Hydrodynamics (answer any two) $2\times10=20$

1. What arrangement of sources and sinks will give rise to complex potential function $W = \log(z - a^2/z)$? also obtain velocity potential, stream function and streamlines.
2. For an irrotational motion in two dimensions, prove that
   $$\left(\frac{\partial \vec{q}}{\partial x}\right)^2 + \left(\frac{\partial \vec{q}}{\partial y}\right)^2 = \vec{q}\nabla q^2$$
   $\vec{q}$ being the velocity vector.

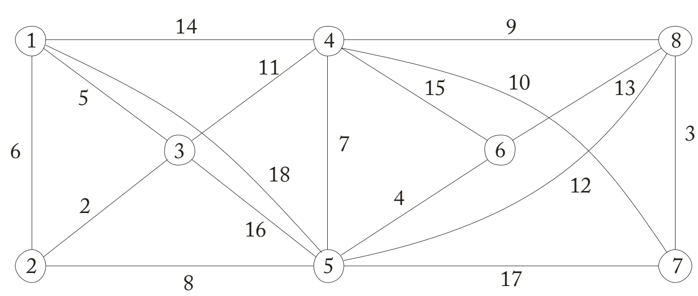
3. A circular cylinder is placed in a uniform stream. Show by using circle theorem that neither a force nor a couple acts on the cylinder.

204. Mathematical Methods (answer any two) 2×10=20

1. a. What is the laplace transform of
   i) Sinh bt?
   ii) Cosh bt?
   b. Find the particular solution of the differential equation
   $y' - 3y' + 2y = 12c^{-2t}$ for which
   $y = 2$ and $y' = 6$ at $t = 0$ (use laplace transform)
2. The temperature U in the semi-infinite rod $o \le x \le \alpha$ is determined by the differential equation
   $$\frac{\partial U}{\partial t} = K\frac{\partial^2 U}{\partial x_2}$$
   Subject to the conditions
   i) $U = o$ when $t = o$, $x \ge o$
   ii) $\frac{\partial U}{\partial x} = -\mu$ (a constant) when $x = o$ of $t > o$
   Making use of the Fourier cosine transform, show that
   $$U(x,t) = \frac{2\mu}{\pi}\int_o^\infty \frac{\cos\lambda x}{\lambda^2}(1 - e^{-k\lambda^2 t})d\lambda$$
3. Using the method of successive approximation, solve the integral equation
   $$\varphi(x) = 2x + \lambda\int_0^1 (x+t)\varphi(t)dt$$
   With $\varphi_0(x) = 1$

205. Operation Research (answer any two) 2×10=20

1. Write down the steady-state solution of the model *{M/M/1:N/FCFS}* and hence show that the average number of customers in the queue is given by
   $$L_q = \frac{\rho^2[1 - N\rho^{N-1} + (N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})}$$
   ρ being the traffic intensity.
2. What is mcant by quadratic programming? Derive, Kuhn-Tucker Conditions for an optimal solution to a quadratic programming problem.
3. Find the minimum spanning tree of the graph G:



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